

June 2022 update

See below the latest updates for the current 20th edition, which are all incorporated in the PDF file of the Kourovka Notebook available at <https://kourovka-notebook.org> (see also <http://math.nsc.ru/~alglog/19tktk.pdf> and <https://arxiv.org/pdf/1401.0300.pdf>).

For convenience of the readers all the updates since the first appearance of the 20th edition are also listed below; the newest comments or solutions added in June 2022 are marked **NEW**.

12.48. Let G be a sharply doubly transitive permutation group on a set Ω (see Archive 11.52 for a definition).

(a) Does G possess a regular normal subgroup if a point stabilizer is locally finite?

(b) Does G possess a regular normal subgroup if a point stabilizer has an abelian subgroup of finite index?

NEW

Comments of 2022: an affirmative answer to part (b) is obtained in permutation characteristic 0 (F. O. Wagner, *Preprint*, 2022, <https://hal.archives-ouvertes.fr/hal-03590818>).

V. D. Mazurov

***19.23.** For a group G , let $\text{Tor}_1(G)$ be the normal closure of all torsion elements of G , and then by induction let $\text{Tor}_{i+1}(G)$ be the inverse image of $\text{Tor}_1(G/\text{Tor}_i(G))$. The torsion length of G is defined to be either the least positive integer l such that $G/\text{Tor}_l(G)$ is torsion-free, or ω if no such integer exists (since $G/\bigcup \text{Tor}_i(G)$ is always torsion-free).

Does there exist a finitely generated, or even finitely presented, soluble group with torsion length greater than 2?

M. Chiodo, R. Vyas

NEW

**Yes, such groups exist (I. J. Leary, A. Minasyan, to appear in *J. Group Theory*, <https://arxiv.org/abs/2112.14546>).*

***19.24.** For a group G , let $\text{Tor}(G)$ be the normal closure of all torsion elements of G . Does there exist a finitely presented group G such that $G/\text{Tor}(G)$ is not finitely presented? Such a group must necessarily be non-hyperbolic.

M. Chiodo, R. Vyas

NEW

**Yes, such groups exist: one soluble example and another virtually torsion-free are constructed in (I. J. Leary, A. Minasyan, to appear in *J. Group Theory*, <https://arxiv.org/abs/2112.14546>).*

NEW

***20.27.** Let G be a finite group, p a prime number, and let $|x^G|_p$ denote the maximum power of p that divides the class size of an element $x \in G$. Suppose that there exists a p -element $g \in G$ such that $|g^G|_p = \max_{x \in G} |x^G|_p$. Is it true that G has a normal p -complement?

A partial answer is in (<https://arxiv.org/abs/1812.03641>). *I. B. Gorshkov*

**No, not necessarily, a counterexample is given by `SmallGroup(192,945)` (B. Sambale, *Letter of 16 February 2022*).*